Name: _____

PRACTICE EXAM (MID-YEAR)

Year 12 Mathematics Methods Exam 2

- Time allowed: 15 minutes reading time
  120 minutes writing time
- CAS calculators may be used.
- Part 1: Twenty multiple-choice questions (20 marks).
- Part 2: Five long-answer questions (50 marks).

Part 1

Multiple-choice questions

Twenty multiple-choice questions
1 mark each: 20 marks
Circle the correct answer.

1 Given $f(x) = \sin^3(3x)$ then $f'(x) =$

A $3\cos^3(3x)$

B $3\sin^2(3x)\cos(3x)$

C $9\sin^2(3x)\cos(3x)$

D $9\sin^3(3x)\cos(3x)$

E $-9\sin^2(3x)\cos(3x)$

2 Given $y = xe^x$ then $\frac{dy}{dx} =$

A $\frac{x}{e} + \frac{1}{e}$

B $2e^x + e$

C e^x

D $-2e^x + 3e$

E $e^x + xe^x$

3 A particle moves according to $x = 1.5t^2 - 6t + 6$. Determine which of the following is incorrect.

A The particle changes direction at $t = 2$.

B The acceleration is constant.

C The velocity is negative for $0 \leq t < 2$.

D The displacement at $t = 2$ is zero.

E The distance travelled for $0 \leq t \leq 3$ is 1.5.

4 Consider a sphere of radius r, volume V and surface area SA. The radius is increasing at a rate of 2 cm/sec.

$\frac{dV}{dt}$ and $\frac{dSA}{dt}$ are equal to:

A $8\pi r^2$ and $16\pi r$

B $4\pi r^2$ and $8\pi r$

C $8\pi r^2$ and $8\pi r$

D $4\pi r^2$ and $16\pi r$

E $\frac{r}{\pi}$ and $\frac{\pi}{r}$

5 The angle of elevation of the top of a building, measured at a distance of 20 m from the base of the building, was written down as 60.5°. If the angle of elevation was actually exactly 60°, determine the error in the calculation of the height of the building.

A 0.72 m

B 0.71 m

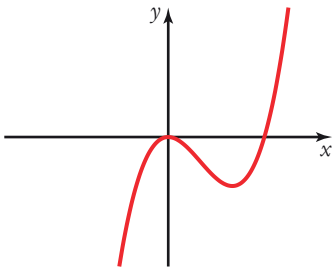
C 34.6 m

D 41.24 m

E 40 m

6 Given $f'(x) = 2x - 3x^2$ and $f(0) = 0$ then the graph of the function $y = f(x)$ is:

A



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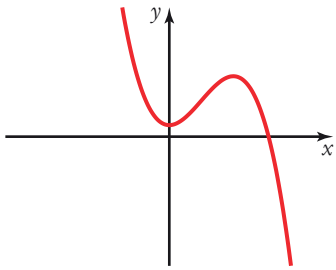

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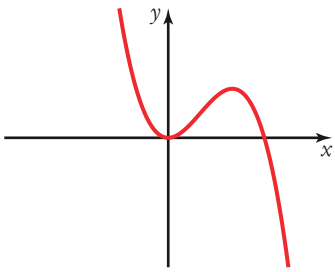

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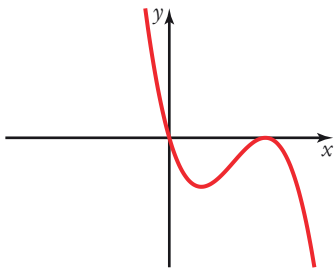

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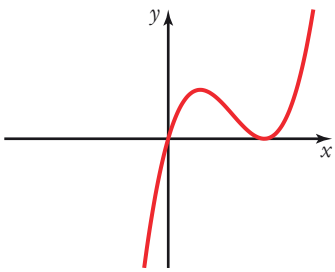


7 Select the statement that is *not* necessarily correct.

A $y = x^3$ has a point of inflection at $x = 0$.

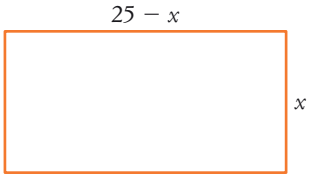
B $y = x^2 - 1$ has x-intercepts at $x = \pm 1$.

C $\dfrac{d^2y}{dx^2} = 0$ at $x = x_1$ means there is a point of inflection at $x = x_1$.

D $\dfrac{dy}{dx} = 0$ and $\dfrac{d^2y}{dx^2} > 0$ at $x = x_1$ means there is a minimum turning point at $x = x_1$.

E $\dfrac{dy}{dx} = 0$ and $\dfrac{d^2y}{dx^2} < 0$ at $x = x_1$ means there is a maximum turning point at $x = x_1$.

8 A piece of string of length 50 cm is formed into a rectangular shape as shown below.



The maximum area, A, occurs when:

A $\frac{dA}{dx} = 0$ and $\frac{d^2y}{dx^2} > 0$

B $\frac{dA}{dx} = 0$ and $\frac{d^2y}{dx^2} < 0$

C $\frac{dA}{dx} = 0$ and $\frac{d^2y}{dx^2} = 0$

D $\frac{dA}{dx} > 0$ and $\frac{d^2y}{dx^2} = 0$

E $\frac{dA}{dx} < 0$ and $\frac{d^2y}{dx^2} = 0$

9 If $P(A) = 0.2$, $P(B) = 0.5$, $P(A \cup B) = 0.6$, which of the following is *not* true?

A $P(A \cap B) = 0.1$

B $P(A \mid B) = 0.2$

C $P(B \mid A) = 0.5$

D A and B are independent

E A and B are mutually exclusive

Use the following information to answer Questions 10 and 11.

The probability that a particular traffic light is red is 0.3. Emily goes through the one traffic light every day on the way to work. Emily works Monday to Friday.

10 What is the probability she only gets the red light on Friday?

A $(0.7)^4(0.3)$

B $(0.7)\ (0.3)^4$

C $(0.7)\ (0.6)\ (0.5)\ (0.4)\ (0.3)$

D ${}^5C_1(0.7)^4(0.3)$

E ${}^5C_1(0.3)^4(0.7)$

11 What is the probability Emily only gets one red light from Monday to Friday?

A $(0.7)^4(0.3)$

B $(0.7)\ (0.3)^4$

C ${}^5C_1(0.7)^4(0.3)$

D ${}^5C_1(0.3)^4(0.7)$

E $(0.7)\ (0.6)\ (0.5)\ (0.4)\ (0.3)$

12 Given $P(A) = 0.5$, $P(B) = 0.4$ and $P(B \mid A) = 0.4$, then $P(A \cup B)$ is:

A 0.2

B 0.3

C 0.4

D 0.7

E 0.9

13 A couple with a newborn baby is chosen at random. If 10% of boys are colour blind and only 5% of girls are colour blind, and there are equal chances of the baby being a boy or a girl, what is the probability that the baby is colour blind?

A 0.075

B 0.025

C 0.05

D 0.15

E 0.925

14 A bag contains 4 doughnuts with pink icing and 2 with green icing. If 2 doughnuts are selected at random, what is the probability of getting 2 doughnuts with different coloured icing?

A $\frac{1}{15}$

B $\frac{8}{15}$

C $\frac{4}{15}$

D $\frac{6}{15}$

E $\frac{2}{5}$

15 Given the function $y = x^2 - \frac{x^4}{2}$, which of the following is *not* correct?

A The function has turning points at $(1, 0.5)$, $(-1, 0.5)$ and $(0, 0)$.

B The function has points of inflection at $x = \pm\frac{1}{\sqrt{3}}$.

C The range of the function is $y \leq \frac{1}{2}$.

D The function has a maximum value at $x = 1$, $x = -1$ and a minimum value at $x = 0$.

E The function is concave up for $x < -\frac{1}{\sqrt{3}}$ and for $x > \frac{1}{\sqrt{3}}$.

16 $\int \sin(x)dx =$

A $\frac{\sin^2(x)}{2} + c$

B $-\frac{\cos^2(x)}{2} + c$

C $-\cos(x) + c$

D $\cos(x) + c$

E $\sqrt{1 - \cos^2(x) + c}$

17 Which of the following is a probability density function?

A

x	1	2	3	4
$P(X = x)$	0.2	0.3	0.4	0.2

B

x	1	2	3	4	5
$P(X = x)$	0.1	−0.3	0.4	0.4	0.4

C

x	−5	−10	−15	−20
$P(X = x)$	0.1	0.2	1.1	0.2

D

x	10	11	12	13
$P(X = x)$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{4}$

E

x	2	4	6	8
$P(X = x)$	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{2}{5}$

18 For the discrete random variable, Z, $E(Z) = 2$ and $E(Z^2) = 4.5$. The standard deviation of Z is equal to:

A $\frac{1}{\sqrt{2}}$

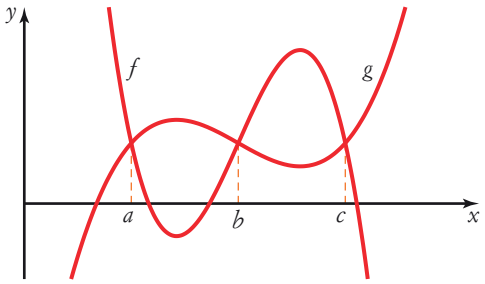
B $\frac{1}{2}$

C 2.5

D $-\frac{1}{2}$

E 4.03

19 The area between the curves shown on the graph below is:



A $\int_a^c \big(f(x) - g(x)\big)dx$

B $\int_a^c \big(g(x) - f(x)\big)dx$

C $\int_a^b \big(g(x) - f(x)\big)dx + \int_b^c \big(f(x) - g(x)\big)dx$

D $\int_a^b \big(f(x) - g(x)\big)dx + \int_b^c \big(g(x) - f(x)\big)dx$

E $\left|\int_a^c \big(g(x) - f(x)\big)dx\right|$

20 $\int_1^4 \sqrt{x}\,dx =$

A $\frac{14}{3}$

B 14

C $-\frac{1}{4}$

D 1

E $\frac{7}{2}$

Part 2

Extended-answer questions

Five extended-answer questions
50 marks
Show your working where appropriate.

Question 1

a Jody had four traffic lights to drive through to get to work. She calculated that the probabilities of getting a red light on traffic lights 1 to 4 as 0.3, 0.4, 0.5 and 0.6.

i Find the probability that Jody had no red lights on the day she was running late.

[2 marks]

ii On another day, Jody was late because she had been held up by the first red light.

Find the probability that on that day she got 4 red lights.

[2 marks]

b i Given $g(x) = e^{-x} + \frac{2}{\sqrt{x}} - \sin(4x)$, determine $g'(\pi)$, leaving your answer in terms of π where appropriate.

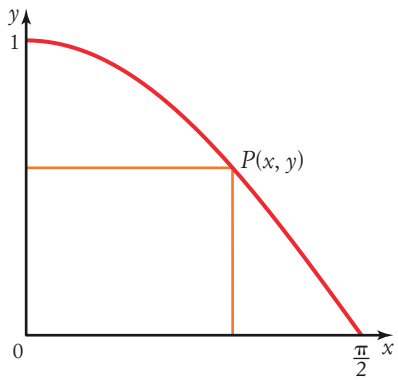
[2 marks]

ii Evaluate $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (\cos(2x) - 1)dx$.

[2 marks]

Question 2

a Find the dimensions of the largest rectangle that can be formed with vertices on the origin, the x-axis, the y-axis and the curve $y = \cos(x)$ for $0 \leq x \leq \frac{\pi}{2}$.



[5 marks]

b Find the area between $y = \cos(x)$ and the x-axis for $-\frac{\pi}{2} \leq x \leq \pi$.

[3 marks]

c In the diagram in part a, P represents a bug travelling along the curve $x = \cos(t)$ for $0 \leq t \leq 2\pi$ where t is time in seconds and x is displacement.

Determine:

i the maximum velocity of the bug for $0 \leq t \leq 2\pi$.

[2 marks]

ii the acceleration of the bug when the displacement is $\frac{1}{\sqrt{2}}$.

[2 marks]

Question 3

Two normal six-sided dice are rolled. When the dice are rolled, it is considered a success if a double six is obtained.

The two dice are rolled 50 times.

a Find the expected values of x, $E(x)$, and the standard deviation of x, $S(x)$.

[3 marks]

b i Construct the sample space for the difference of the two dice.

[2 marks]

ii Construct the probability density function for the difference in the two dice.

[2 marks]

iii Find the expected difference when two dice are rolled.

[2 marks]

iv When two dice are rolled, what is the most likely difference?

[1 mark]

Question 4

Judy and Jimmy are sitting a test consisting of 20 multiple-choice questions.

a Judy had studied hard and estimated she would be able to answer 15 of the multiple-choice questions correctly. She would have to guess the last five. There are five possible answers to each question.

What is the probability that Jody got 17 questions correct?

[3 marks]

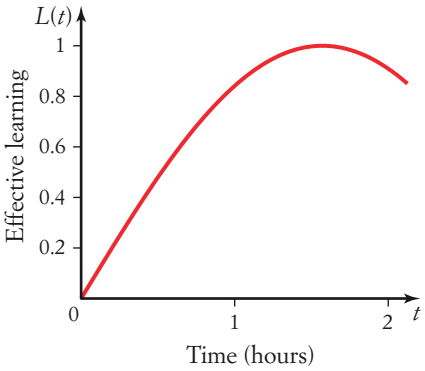
b Jimmy didn't study much and he estimated he would be able to answer 7 of the multiple choice questions correctly. To obtain a pass in the test, it was necessary to get at least 10 questions correct.

What is the probability that Jimmy passed?

[3 marks]

Jimmy's mother was concerned about the amount of study time Jimmy did.

c Jimmy's learning ability during study time can be represented by the function $L(t) = \sin(t)$ for $0 \leq t \leq 2$, i.e. it drops off about $t \approx 1.6$ hours.



Jimmy is happy if he can study for one hour, but his mother wants him to do 15 mins more, i.e. $\delta t = 0.25$ at $t = 1$.

Find the percentage increase in effective learning given the 15 minutes extra study.

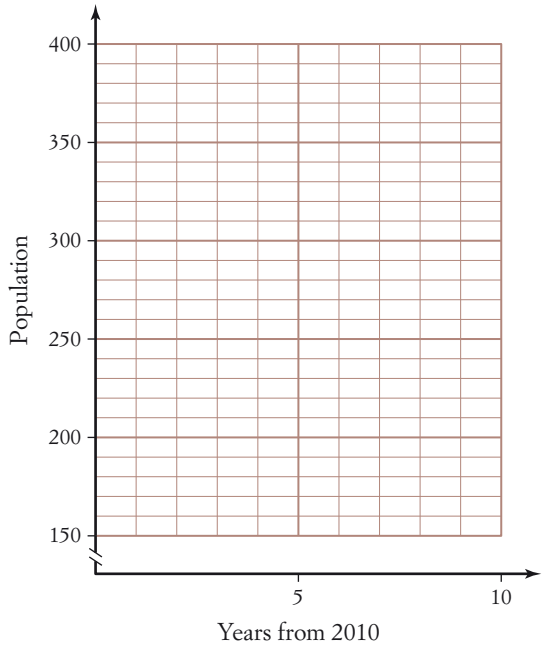
[4 marks]

Question 5

In January 2010 a population of wild cats was found in Pompeii.
The cats were studied and it was determined that their population was modelled by the equation $P_{wc} = 200e^{0.08t}$ where t is in years from January 2010.
It is known that rats also lived in Pompeii and that in 2005 they had been subjected to poisoning, but it was slow to take effect.
In January 2010 it was determined that the population of rats could be modelled by the equation $P_r = 400e^{-0.04t}$.

a Sketch the functions representing the populations of the wild cats and the rats on the set of axes below.

[3 marks]



b In what year is the rat population expected to be less than 100?

[2 marks]

c i Find the rate of growth of the populations of wild cats and of rats.

[2 marks]

ii Which population is changing at the fastest rate? Explain.

[1 mark]

If the cat population equalled the rat population, the rats would die at a faster rate.

d When is this expected to happen?

[2 marks]